



Rossmoyne Senior High School

Semester One Examination, 2024

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section One: Calculator-free

SOLUTIONS

WA student number: In figures

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Circle your teacher:

Ms Alvaro Ms Lee
Mr Liu Ms Robinson

In words

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	93	65
Total					100

Instructions to candidates

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
1	5	
2	9	
3	7	
4	8	
5	9	
6	7	
7	7	
Section 1 Total	52	
Weighted ($\times 0.7143$)	35%	
Section 2 Weighted	65%	
Total	100%	

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Section One: Calculator-free

35% (52 Marks)

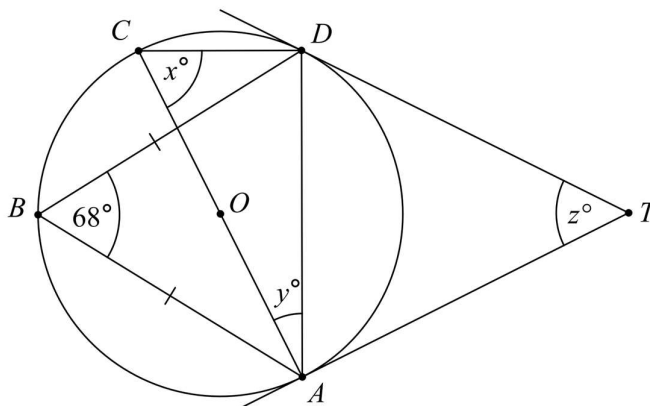
This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(5 marks)

In the diagram below, $BD = BA$, $\angle DBA = 68^\circ$ and DT and AT are tangents to the circle centred at O .



Determine, *giving reasons*, the value of:

(a) x

(1 mark)

Solution	
$x = 68^\circ$ (angles in the same arc)	
Specific behaviours	
✓	States the value of x with a valid reason.

(b) y

(2 marks)

Solution	
$\angle CDA = 90^\circ$ (angle in semicircle)	
$y = 90 - 68 = 22^\circ$	
Specific behaviours	
✓	States or shows $\angle CDA = 90^\circ$ with a valid reason.
✓	States the value of y .

(c) z

(2 marks)

Solution	
$\angle DAT = 90 - 22 = 68$	
(tangent $\perp$ radius)	
$z = 180 - 2 \times 68 = 44^\circ$	
(tangents from a point are congruent)	
Specific behaviours	
✓	Determines $\angle DAT$.
✓	Determines the value of z with a valid reason.

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Question 2

(9 marks)

- (a) A student wants to prove that the following statement is true for all integers, x :

If x^2 is even, then x is even

To prove that this statement is true, it is sufficient to prove which of the following:

- A** If x^2 is odd, then x is odd **B** If x is odd, then x^2 is odd
C If x is even, then x^2 is even **D** If x is odd, then x^2 is even

Explain how you made your choice. You are **not** required to prove it is true. (3 marks)

Solution
Statement B , This statement is the contrapositive, if the contrapositive is true, then the original statement is also true.
Specific behaviours
✓ Chooses correct statement. ✓ Uses the correct terminology to describe their statement. ✓ Explains why proving their chosen statement is true, proves the original statement is true.

- (b) A set contains n distinct positive integers. Prove that at least two integers in this set have the same remainder when divided by $n - 1$. (3 marks)

Solution
Possible remainders are: 0, 1, 2, 3, ..., $n - 2$ There are $n - 1$ possible remainders (the pigeonholes) and n integers (the pigeons). As the number of pigeons > number of pigeonholes, at least two integers have the same remainder.
Specific behaviours
✓ Determines pigeons and pigeonholes ✓ Determines the possible remainders and there are $n - 1$ remainders. ✓ Explains using pigeonhole principle why at least two integers have the same remainder.

- (c) Determine the truth of the following statement, where m , n and p are integers. Explain how you made your decision. (3 marks)

$$\forall n \forall m \exists p: p = \frac{m+n}{2}$$

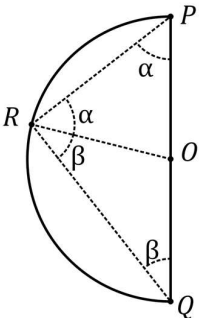
Solution 1
Statement is false. If m and n are different parities, then $m + n$ is odd, hence $\frac{m+n}{2}$ is not an integer.
Specific behaviours
✓ States statement is false. ✓ Considers the parity of m and n. ✓ Explains why p is not an integer when m and n are different parities.
Solution 2
If $m = 1, n = 2$ then $p = 1.5$ However, p is not an integer. Hence statement is false.
Specific behaviours
✓ Determines a suitable counter-example. ✓ Shows that p is not an integer. ✓ Concludes statement is false.

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Question 3

(7 marks)

- (a) Consider any point R that lies on the circumference of a circle with centre O and diameter PQ . Using only triangle properties, prove that $\angle PRQ = 90^\circ$. (3 marks)

Solution	
	Consider $\triangle OPR$ in which $OP = OR = \text{radius of circle}$, and so this triangle is isosceles, with $\angle OPR = \angle ORP = \alpha$.
	Likewise, $OQ = OR$ and so $\triangle OQR$ is also isosceles, with $\angle OQR = \angle ORQ = \beta$.
	Angles in $\triangle PQR$ are $\angle QPR = \alpha$, $\angle PRQ = \alpha + \beta$ and $\angle PQR = \beta$.
	Hence $\alpha + \alpha + \beta + \beta = 180^\circ \Rightarrow \alpha + \beta = \angle PRQ = 90^\circ$.
Specific behaviours	
✓ uses radius of circle to explain why $OP = OR = OQ$ ✓ uses isosceles triangles to explain why $\angle OPR = \angle ORP$ and $\angle OQR = \angle ORQ$ ✓ uses angle sum of triangle to complete proof	

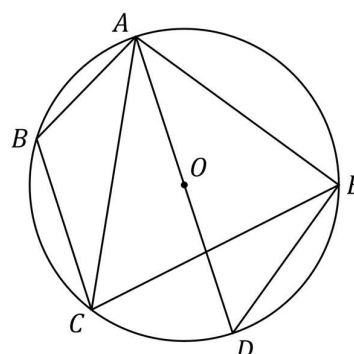
- (b) Points A, B, C, D and E lie on circle with centre O and diameter AD as shown in the diagram to the right (not to scale).

$$\angle BAC = 25^\circ.$$

$$\angle BCA = 18^\circ.$$

$$\angle ACE = 54^\circ.$$

Determine the size of the following angles:



- (i) $\angle OAE$.

(2 marks)

Solution	
$\angle ADE = \angle ACE = 54^\circ$	
$\angle OAE = 90^\circ - \angle ADE = 90^\circ - 54^\circ = 36^\circ$	
Specific behaviours	
✓ obtains $\angle ADE$ ✓ correct size of angle	

- (ii) $\angle CED$.

(2 marks)

Solution	
$\angle AEC = \angle BEA + \angle BEC = \angle BCA + \angle BAC = 25^\circ + 18^\circ = 43^\circ$	
$\angle CED = 90^\circ - \angle AEC = 90^\circ - 43^\circ = 47^\circ$	
Specific behaviours	
✓ obtains $\angle AEC$ ✓ correct size of angle	

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Question 4

(8 marks)

- (a) State, with justification, whether the vectors $\begin{pmatrix} 4 \\ 24 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$ are parallel, perpendicular or neither. (1 mark)

Solution
Perpendicular, since $\begin{pmatrix} 4 \\ 24 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \end{pmatrix} = 24 - 24 = 0$.
Specific behaviours
✓ correct response, with justification

- (b) Determine the vector projection of $\begin{pmatrix} 2 \\ 14 \end{pmatrix}$ on $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$. (2 marks)

Solution
$\frac{\begin{pmatrix} 2 \\ 14 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \end{pmatrix}}{\begin{pmatrix} 3 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -4 \end{pmatrix}} \times \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \frac{6 - 56}{9 + 16} \times \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$
Specific behaviours
✓ indicates correct method ✓ correct vector

- (c) Determine the value(s) of the constant μ given that vectors $\begin{pmatrix} 6 \\ \mu \end{pmatrix}$ and $\begin{pmatrix} -2 \\ \mu - 1 \end{pmatrix}$ are perpendicular. (2 marks)

Solution
$\begin{pmatrix} 6 \\ \mu \end{pmatrix} \cdot \begin{pmatrix} -2 \\ \mu - 1 \end{pmatrix} = \mu^2 - \mu - 12 = 0$ $(\mu + 3)(\mu - 4) = 0$ $\mu = -3, \quad \mu = 4$
Specific behaviours
✓ correct scalar product ✓ correct values of μ

- (d) Evaluate $(\vec{r} - 4\vec{s}) \cdot (3\vec{s} + 2\vec{r})$ given that $\vec{r} \cdot \vec{s} = -10$ and vectors $\vec{r}$ and $\vec{s}$ have magnitudes 4 and 5 respectively. (3 marks)

Solution
$\begin{aligned} (\vec{r} - 4\vec{s}) \cdot (3\vec{s} + 2\vec{r}) &= 3\vec{r} \cdot \vec{s} + 2\vec{r} \cdot \vec{r} - 12\vec{s} \cdot \vec{s} - 8\vec{s} \cdot \vec{r} \\ &= 2 \vec{r} ^2 - 12 \vec{s} ^2 - 5\vec{r} \cdot \vec{s} \\ &= 2(4)^2 - 12(5)^2 - 5(-10) \\ &= 32 - 300 + 50 \\ &= -218 \end{aligned}$
Specific behaviours
✓ correctly expands using scalar products ✓ simplifies product using $\vec{a} \cdot \vec{a} = \vec{a} ^2$ ✓ correct value

Question 5

(9 marks)

Trolleys at a shopping centre are collected from a car park using a trailer. The trailer can hold a maximum of five trolleys.

Trolleys can come from any of four stores: Aldo, Coldo, The Project Shop or W-Mart. Each store's trolleys are different from each other, but all trolleys from the same store are identical.

(a) How many ways can five trolleys be put on the trailer if

- (i) two Coldo shopping trolleys have been rammed together and cannot be separated, and there is one trolley from each of the other three stores? (1 mark)

Solution
$4! = 24$
Specific behaviours
✓ Recognises there are four 'trolleys' to be arranged and determines the number of arrangements.

- (ii) there are three trolleys from The Project Shop and two from W-Mart? (2 marks)

Solution
$\frac{5!}{3!2!} = 10$
Specific behaviours
✓ Arranges the five trolleys.
✓ Divides by $3!2!$ and determines final answer.

- (iii) if there is at least one trolley from each store? (3 marks)

Solution
$\frac{4! \times 5 \times 4}{2!} = 240$
Specific behaviours
✓ Arranges a trolley from each store.
✓ Recognises five places for a second trolley to be placed, and multiplies by four possible trolleys.
✓ Divides by two due to repetition and determines final answer.

People need to pay to use the Aldo trolleys. To get a refund, each Aldo trolley needs to be chained to another Aldo trolley. Hence, Aldo trolleys are often chained together.

- (b) How many ways can five trolleys be put on the trailer if there are **at least two** consecutive Aldo trolleys? (3 marks)

Solution
Spots 1 & 2: $1 \times 1 \times 4 \times 4 \times 4 = 64$
Spots 2 & 3: $3 \times 1 \times 1 \times 4 \times 4 = 48$ (first can't be Aldo)
Spots 3 & 4: $4 \times 3 \times 1 \times 1 \times 4 = 48$ (first can be Aldo, but second can't)
Spots 4 & 5: $(4 \times 4 - 1) \times 3 \times 1 \times 1 = 45$ (eliminating AA in 1 & 2 already done)
Total arrangements = 205
Specific behaviours
✓ Splits the arrangement of trolleys satisfying the condition into manageable subsets.
✓ Recognises when the first trolley can and can't be an Aldo trolley.
✓ Determines correct answer by combining these subsets.

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Question 6

(7 marks)

Points A and B have position vectors $\vec{a} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

- (a) Determine the vector $\vec{AB}$.

(1 mark)

Solution
$\vec{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$
Specific behaviours
✓ correct vector

- (b) Determine the vectors $\vec{r}$ and $\vec{s}$ given that $\vec{a} = \vec{r} + 2\vec{s}$ and $\vec{b} = 2\vec{r} - \vec{s}$.

(3 marks)

Solution
First equation added to $2 \times$ second equation gives $\vec{a} + 2\vec{b} = 5\vec{r}$. Hence: $\vec{r} = \frac{1}{5} \left(\begin{pmatrix} -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 8 \\ -6 \end{pmatrix} \right) = \begin{pmatrix} 7/5 \\ -1 \end{pmatrix}$ $\vec{s} = 2\vec{r} - \vec{b} = \begin{pmatrix} 14/5 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} -6/5 \\ 1 \end{pmatrix}$
Specific behaviours
✓ uses equations to eliminate one unknown vector ✓ obtains $\vec{r}$ ✓ obtains $\vec{s}$

- (c) Determine the value(s) of the constant μ for which $|\vec{b} - \mu\vec{a}| = \sqrt{13}$.

(3 marks)

Solution
$\vec{b} - \mu\vec{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} - \mu \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 + \mu \\ -3 - \mu \end{pmatrix}$ $(4 + \mu)^2 + (-3 - \mu)^2 = 13$ $16 + 8\mu + \mu^2 + 9 + 6\mu + \mu^2 = 13$ $2\mu^2 + 14\mu + 12 = 0$ $\mu^2 + 7\mu + 6 = 0$ $(\mu + 6)(\mu + 1) = 0$ $\mu = -1, \mu = -6$
Specific behaviours
✓ forms equation for μ ✓ obtains quadratic equation equal to 0 ✓ factors to obtain correct values

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Question 7

(7 marks)

- (a) A multiple-choice test consists of 4 questions, each with 5 possible answers A, B, C, D and E . Determine the least number of students who must take the test to ensure that there will be at least 3 students with identical sets of answers. Justify your answer. (3 marks)

Solution
Number of different ways test can be answered is $5^4 = 625$ (Pigeon Hole) $M \times 625 + 1$, at least 3 students with identical answers. $m+1 = 3$, $m = 2$ Using the pigeon-hole principle, if 2 students complete each of the 625 ways then $2 \times 625 = 1250$ students take the test. If one more student takes the test, then there will be at least 3 students with identical sets of answers. Hence the least number is 1251 students.
Specific behaviours
✓ Identify the Pigeon Hole -correct number of different ways to answer test ✓ apply correct formula to calculate correct least number of students ✓ justifies answer

- (b) Consider the identity from Pascal's triangle $5 \binom{n}{5} = (n-4) \binom{n}{4}$.

- (i) Show that the identity is true when $n = 5$. (1 mark)

Solution
LHS = $5 \binom{5}{5} = 5 \times 1 = 5$, RHS = $(5-4) \binom{5}{4} = 1 \times 5 = 5$ and so identity is true.
Specific behaviours
✓ clearly substitutes and then simplifies for each side of identity

- (ii) Use the definition $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ to prove the identity for $n \in \mathbb{Z}, n \geq 5$. (3 marks)

Solution
$ \begin{aligned} \text{RHS} &= (n-4) \binom{n}{4} \\ &= \frac{(n-4)n!}{4!(n-4)(n-5)!} \\ &= \frac{n!}{4!(n-5)!} \\ &= \frac{5 \times n!}{5!(n-5)!} \\ &= 5 \binom{n}{5} = \text{LHS} \end{aligned} $
Specific behaviours
✓ expresses RHS using factorials ✓ correctly eliminates $n-4$ factor ✓ correctly obtains LHS

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Supplementary page

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